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Solar Force Modeling of Block IIR Global Positioning System Satellites

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Scientific applications of the Global Positioning System require that the space vehicles be located with an accuracy of a few centimeters. The most important uncertainties in position estimation are the result of direct and indirect solar forces. Perhaps as early as late 1996, Block IIR space vehicles will begin to replace the existing Blocks II and IIA. We give formulas for the solar force to be expected on Block IIR and evaluate their probable accuracy based on our previous experience. These a priori formulas include indirect solar forces, including the reradiation of sunlight in the form of heat from the space vehicle's body and solar panels, but do not include radiation-induced outgassing, especially from the multilayered insulation that wraps the space vehicle body. We discuss the best way to combine these a priori models with real-time tracking data to optimize ephemeris accuracy.

Introduction

AMONG the many unique features of the Global Positioning System (GPS), one fact is of primary importance for its use in geodesy, studies of crustal dynamics, and support of such space missions as TOPEX/Poseidon: it is the first system of navigational satellites in which errors in knowledge of the Earth's gravitational field have negligible effect, and orbit accuracy is limited almost entirely by errors in modeling the force of solar radiation on the space vehicles (SVs). These SVs are high altitude (semimajor axes = 26,560 km = 4.16 Earth radii) and relatively insensitive to the higher-order gravitational harmonics, which are well known in any case via satellites such as LAGEOS. But GPS SVs have a large cross-sectional area (about 13.6 m^2 for Block II/IIA) and are accelerated about 1×10^{-7} m/s by direct solar radiation. Table 1 shows our calculation (using the Aerospace TRACE program) of perturbations on a typical GPS orbit over 12 h. After the Earth oblateness J2, the sun, and the moon, solar radiation is the most important.

An a priori model calculates the solar force to be expected on the vehicle as a function of its orientation and of its cross-sectional area and optical properties. Direct solar pressure can be pictured as the net momentum imparted to the SV by photons striking and recoiling from its opaque surfaces. Indirect solar pressure is caused, for example, by heat absorbed and reradiated from body surfaces and by outgassing, whereby solar energy and momentum is returned to space by volatile materials on and in the SV body. Indirect forces are of two kinds: predictable, such as the effects of Earthshine and SV heat flow; and anomalous, including outgassing and the so-called Y-bias force. Because none of the indirect effects, predictable or anomalous, were included in the early computer software developed for GPS, many scientific GPS analysts have bypassed the modeling problem entirely and have determined SV accelerations directly as stochastic parameters to be estimated by tracking data.[1,2] To attain highest accuracy, some such real-time estimation is necessary. Nevertheless, good a priori models are indispensable for three reasons: 1) many applications require not just precise fitting of orbits to tracking data already taken, but orbit prediction; 2) by using a standard force model, workers more readily can intercompare ephemerides, which are calculated by many different agencies from data taken all over the world using different techniques; and 3) to filter real data accurately and reliably, one should know the statistics of the parameters measured—how fast they usually change, and how fast they can change—and this requires knowledge of the physics of the problem, and therefore, good a priori modeling. Our intention in publishing standard models is not to force all workers to use the same set of constants, but to provide a basis of intercomparison to which improved techniques can be benchmarked.

More specifically, our strategy is to identify a priori the most crucial parameters, and so determine the form of the function that the acceleration history should fit. Then the user can (when necessary) use tracking data to improve those parameters and to pinpoint anomalies. We apply that strategy to the existing Block II/IIA solar force model, and we predict the expected formulas and behavior for the forthcoming Block IIR.

Existing a Priori GPS Models of Solar Force

All GPS solar force models are either applications or revisions of the formulas supplied by Rockwell International, the spacecraft contractor for the GPS Block I and II/IIA SVs.[3] Of these, at the time this is written, only one Block I SV remains healthy, and the entire GPS constellation consists of Block II/IIA vehicles. The Rockwell document gave the dimensions and optical properties of every major SV surface except one, and, for each surface, stated the special equation for the force of visible sunlight on that surface. The IBM Federal Systems Division coded these formulas in computer programs called ROCK4 (for Block I) and ROCK42 (for Block II), and The Aerospace Corporation made minor corrections. However, on Block II, because of a late redesign of the SV, a major surface was not included in the Rockwell document or in ROCK42: a plume shield that screens the solar panels from the effluent of the apogee thrust engine. Furthermore, the heat radiated from the SV body surfaces, which is not included in ROCK4 or ROCK42, is a substantial fraction of the total solar force (for Block II/IIA, 5.0%), and the Rockwell formulas must be corrected to include this.[4] Finally, the cumbersome piece-by-piece computer code of ROCK4/ROCK42 is not necessary in an operational program, because the output is a function of one argument (the angle B between the Z axis and the sun) and can be represented with high precision by a Fourier series in that angle. For all of these reasons, we recommended[5] that scientific workers discontinue use of ROCK4 and ROCK42 and use these Fourier series. For Block II/IIA, we called the formulas T20 (T to signify that thermal radiation is included, 2 for Block II, and 0 for

Table 1 Typical perturbations on a GPS SV after one revolution (12 h)

Perturbation	Magnitude, m
J2	15,600
Moon	880
Sun (gravitation)	420
Solar radiation	130
C (2,2), S (2,2)	120
C (3, m), S (3, m)(all m)	20

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the zeroth release, to allow for later revisions). In SV body coordinates, in which the Z axis points toward the Earth center and the SV is maneuvered to keep the sun in the XY plane, the T20 formulas for the X- and Z-force components are, in units of 1×10^{-5} N,

$$\begin{aligned} X &= -8.96 \sin B + 0.16 \sin 3B + 0.10 \sin 5B - 0.07 \sin 7B \\ Z &= -8.43 \cos B \end{aligned} \tag{1}$$

These formulas are the Fourier transform of an amended version of ROCK42, in which corrections are made for the plume shield, and the basic equations for solar force are revised to approximate the thermal reradiation from the SV body. (The revised equations are derived in Ref. 5 and discussed in Ref. 6; the physics that underlie the basic equations are discussed in Ref. 7.) We retained all terms with absolute magnitude larger than 1×10^{-7} N; because of the symmetry of the SV body and the approximation used for the plume shield, there is only one such term in Z. There is no a priori Y force component because nominally the SV always is maneuvered to keep the sun in the XY plane of symmetry and because the Block II/IIA SVs are designed to vent heat equally out of both $+Y$ and $-Y$ body radiators. However, because of vehicle misalignments, there is an anomalous force called Y bias (to be discussed). T20 is at present an International Earth Rotation Service standard model[8] and as such may enter some of the products of the International GPS Service (IGS).

How good is T20? In Ref. 5, we itemized the approximations and uncertainties in the physics of the model and estimated that the total predicted force might be in error by as much as 3%. A test of this prediction is provided by a recent study[7] that measured the ratio of actual SV acceleration away from the sun to that predicted by an a priori model, for all operational GPS vehicles (see Table 2). Two weeks of data, from July 25, 1993, to Aug. 7, 1993, from the five GPS operational control system (OCS) monitor stations were compared to the Defense Mapping Agency (DMA) precise ephemerides for the same periods. Using the DMA determinations, the OCS data were screened for clock jumps, and three days of OCS data were determined to be clean enough to extract precise values of solar force. Three models were intercompared: ROCK42, which still is an official Air Force standard; T20; and the Aerospace Generalized Solar Force Model (GSFM), which models major SV body surfaces and the solar space panels as either flat plates or cylinders, and includes antenna structures as cylinders, performing simple shadowing calculations.

The PRN (satellite number) 02 clearly is anomalous, probably a result of an incorrect value of the SV mass in the OCS database. For the other 16 vehicles, as expected, the error in ROCK42 resulting from the neglected thermal radiation and the plume shield produce a root mean square (rms) error of 11.0%. The simplified Aerospace GSFM model has approximately the error we predicted for T20: rms error equals 2.6%. The T20 model performed unexpectedly well, with an rms error of 1.1%. We believe that there must have been a fortuitous cancellation of errors in T20, and that the performance of GFSM is what is to be expected of a T20-like model.

Table 2 Fractional error in mean solar force (deviation from unity)

PRN number T20	ROCK42 GSFM	T20	GSFM
01	0.115	0.012	0.057
02	0.212	0.097	0.111
07	0.120	0.010	0.026
14	0.117	0.008	0.023
16	0.111	0.003	0.018
17	0.105	0.000	0.012
18	0.077	−0.023	0.019
19	0.114	0.004	0.022
20	0.118	0.011	0.025
22	0.122	0.015	0.028
24	0.118	0.012	0.022
25	0.112	0.002	0.021
26	0.087	−0.014	0.029
27	0.110	0.001	0.019
28	0.110	0.001	0.017
29	0.087	−0.014	0.029
31	0.119	0.009	0.026
(rms error: all SVs)	0.110	0.011	0.026

In actual orbit determination, however, the errors in the mean solar force will be removed for all models when a scale factor is estimated from several days of tracking data. Nevertheless, orbit determination benefits from an accurate physical model, because it gives the correct shape of the force function over the 12-h period of revolution. This shape is difficult to deduce from pseudorange data alone, because it aliases easily into errors from other sources and because it is not practical to tune a Kalman filter to respond to signatures of such a short period. The small mean error of T20 encourages us to believe that the short-term behavior, represented by the several terms of the Fourier series, also is accurate. However, the overall accuracy to be expected in using T20 depends on how well the anomalous solar forces can be either modeled or estimated.

Anomalous Forces

Early in the GPS program, several workers reported unmodeled along-track accelerations, different for each SV, and apparently produced by a force in the body-fixed $+Y$ or $-Y$ direction, that is, along the nominal axis of the solar panel center beams. It was shown that misalignments of these beams from the nominal direction, and/or misalignments of the solar sensors, of about 0.5–1 deg would account for the data,[5] and The Aerospace Corporation recommended solving for Y bias via filtered tracking data as a parameter having about 1% of the magnitude of the total solar force and a decay time of 5–20 weeks. For any SV, Y bias should be proportional to the sine of the total misalignment angle (sometimes called yaw angle), but it is not included in any a priori model, because the yaw angle is not known in advance.

During and after an eclipse, misalignments are more severe. At the 1992 Spring Annual Meeting of the American Geophysical Union, we reported the seek-and-find behavior of the solar panels of a typical SV after eclipse: the panels began to turn toward the sun as soon as it appeared, but then the midnight turn maneuver occurred, and the panels went back the way they came until finally aligning with the sun after about 20 min. A subsequent study[9] showed that there was no one pattern for all vehicles. During eclipse, when the sun sensors provide no input, the attitude control system (ACS) was driven by noise in the electrical system, so that the rate of turn was a random variable. Following a recommendation of this study, GPS controllers now bias the ACS by a fixed signal that overrides the noise, so that the yaw rate has the maximum permitted value in a fixed direction. Since this maximum yaw rate varies over each vehicle's history with mass change and momentum dumping, the Jet Propulsion Laboratory (JPL) finds it necessary to estimate this rate for every eclipse and every noon turn. The forthcoming Block IIR satellites will use a different attitude management strategy in which yaw rate will be suppressed for beta angles of less than 1.5 deg. Whether this will eliminate all problems remains to be seen.

Another anomaly is, we believe, the result of outgassing. Soon after arriving on orbit, the SV's actual acceleration exceeds the T20 predicted value by up to 8 or 9% and then decays in roughly exponential fashion. Table 3, taken from DMA plots of radiation pressure

Table 3 Exponential decay of the anomalous force (outgassing?)

Space vehicle number	Initial anomaly (fraction total force)	Decay time (1/e), week
13	0.04	12
14	0.03	12
15	0.04	10
16	0.08	10
17	0.06	15
18	0.02	20
19	0.07	20
20	0.04	20
21	0.08	10
24	0.05	20

scale estimates, characterizes approximately the size and decay time of this anomaly.

In Ref. 5, we suggested that this anomaly was the result of outgassing, perhaps residual gas from the apogee engine. Subsequently, we have examined four candidate sources of gas: 1) the solar panels, 2) the apogee engine, 3) the SV body, and 4) the multilayered insulation (MLI) that wraps the SV body. The most important clue is the magnitude of the impulse represented by the numbers in Table 3: from 20 to 75 kg-m/s. The mass of gas required to produce a given impulse is inversely proportional to the molecular speed; therefore, it varies with the square root of the molecular weight divided by absolute temperature. If the material is air (molecular weight 29) and the temperature is 50°C (the solar panels), 0.060 to 0.225 kg of gas would be required, or about 50–200 liters of air at standard pressure and temperature. The whole volume of the aluminum honeycomb in the Block II/IIA solar panels is 206 liters, and almost all of the air therein is vented before reaching orbit. The total mass of adhesives is about 2–3 kg, and will not outgas as much as 10% of their weight. Furthermore, gas from the solar panels will not expand preferentially toward the sun, because the volatile materials are on the other side of the solar cells and the cover glass. Therefore, the solar panels can be ruled out. The apogee engine would apply its force in the $+Z$ direction in body-fixed coordinates. It would not alias into the solar force to the same degree for all vehicles.

A stronger argument applies against the SV body: Block IIA was redesigned from Block II by closing most of the orifices; nevertheless, no difference in the size or duration of the anomalous force can be perceived between Blocks II and IIA. Therefore, we are left with one promising candidate for outgassing: the MLI insulation. Its effluent is expected to be about 99% water,[10] an efficient propellant (molecular weight 18). Masses of 40–150 g of water would account for the data of Table 3. The effluent is ejected by the heat of the sun and would be expected to impart a recoil to the SV away from the sun, in agreement with the data. We believe that the exponentially decaying anomalous force is due to the MLI and, because this is a standard feature of spacecraft design, should affect many satellites alike. At the meeting of International Association of Geodesy Special Study Group 2.130 in May 1993, it was noted that TOPEX showed very much the same initial solar force anomaly as GPS. We conclude that the forthcoming Block IIR replacement SVs will repeat the history of the Block II/IIAs.

Block IIR Solar Force Model

From the Block II/IIA experience just summarized, we can specify what the general form of a useful Block IIR a priori solar force model should be, what accuracy it can reasonably attain, what warnings should be placed upon its use, and how it can be combined optimally with real-time tracking data in the orbit determination process. We are justified in seeking an accuracy of 2–3%, for a model to be used outside eclipse and for SVs that have aged long enough (about 6 months) to be free of outgassing effects. The model should be a function of the one angle that varies appreciably: the angle between the sun and the $+Z$ axis (the navigation antennas), which the Block IIR contractor, Lockheed Martin, calls the azimuth angle. It will consist, therefore, of formulas for the X and the Z components of force. The Y force will be approximately equal to the total force times the sine of the misalignment angle (the elevation angle), but one must solve for this from tracking data. Fourier series in the azimuth angle are appropriate and have the advantage that the coefficients of the series can be adjusted by those agencies (e.g., the JPL, the IGS) that have the means and the need to do so. Because the shape of the function, i.e., its variation over the 12-h orbital period, cannot be corrected as readily from tracking data as can the average magnitude, terms should be retained down to about 1% of the total force.

For this purpose, the relevant facts about the Block IIR satellite are these (also see Fig. 1).

1) The SV body is approximately a rectangular box, with a cylindrical shroud to cover the main antenna assembly that points toward the Earth and a plume shield structure flush with the $-Z$ surface. We thereby avoid the complications encountered with Block II/IIA, of modeling its exposed navigation antennas and its flaring conical plume shield.

Table 4 Areas and optical parameters of GPS Block IIR surfaces

SV component	Area, m^2	ν, reflectivity	μ, specularity
Solar panels (each of four)	3.40	0.28	0.85
Exposed panel beams (each of two)	0.16	0.85	0.85
$+$ and $-Z$ faces	3.75	0.06	0
$+$ and $-X$ faces	3.05	0.06	0
Antenna shroud	0.89	0.06	0
Plume shield	0.17	0.06	0
W-sensor elements			
Width	0.007	0.06	0
Breadth	0.024	0.06	0
Lengths (low band)			
Element 1	1.66	0.06	0
Element 2	1.86	0.06	0
Element 3	2.42	0.06	0
Element 4	2.97	0.06	0
Element 5	3.40	0.06	0
Lengths (high band)			
Element 1	0.77	0.06	0
Element 2	0.91	0.06	0
Element 3	1.07	0.06	0
Element 4	1.24	0.06	0
Element 5	1.47	0.06	0
Element 6	1.58	0.06	0

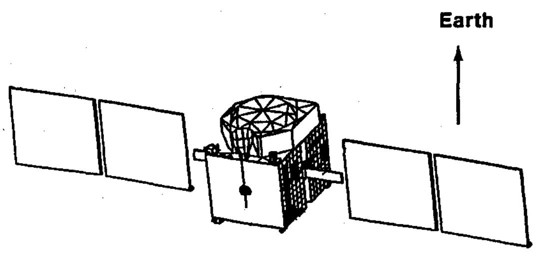

Fig. 1 GPS Block IIR space vehicle.

2) The SV body is almost black; the reflectivity is quoted to be 6%. Thus, the distinctions between specular and diffuse reflection and the different treatments of flat vs cylindrical surfaces (see Ref. 5) are not important. The cross-sectional area of the SV silhouette almost entirely determines the direct solar force. On the other hand, the calculation of heat flow, of energy either absorbed from the SV body surface or reradiated from it, becomes very important.

3) There are four large solar panels, two on each side of the body, each 1.77×1.92 m, suspended on booms that have a cross section 0.16 m^2 on each side not shielded by the panels. The panels plus booms account for about 80% of the maximum force along the X axis and about 75% along the Z axis.

4) The only irregular structures protruding from the SV body that are large enough to need special treatment are the W-sensor antennas, not shown in Fig. 1: the large low-band antenna on the $-X$ side and the smaller high-band antenna on $+X$. The combined cross section of all elements is about 0.5 m when viewed along $+Z$, but they are black and vanish when seen against the SV body.

5) Unlike Blocks I and II/IIA, the IIR SVs will experience a full range of 360 deg in the elevation angle (sun to $+Z$ axis). There will be no noon or midnight turn.

The dimensions and optical parameters relevant to the solar force model are given in Table 4. From these properties, we calculated the solar force for the 0–360 deg range of angles between the sun and $+Z$, using the logic of Ref. 5 and the GSFM model of Ref. 7. For the thermal coefficient (rho) required in the GSFM, which is the fraction of radiation striking an SV body surface that will be reradiated by that surface as heat, we adopted the value of 0.9 as provided by Lockheed Martin.

The solar panels require special treatment. Most of the heat received by the front surface passes through and is reradiated from the dark surface. Because the panels are so large, exact calculation of the heat flow is important.[5,11] The thermal properties of the layer of materials in the solar panels are shown in Table 5.

Table 5 GPS solar array material thermal properties

Layer	Material	Thickness, 0.001 in.	Thermal, W/in. °C	Conductivity, W/(m²K)
1	Coverglass	10	0.036	1.417
2	DC 93-500	2	0.004	0.158
3	Solar cell	8	3.81	150.0
4	RTV-566	3	0.004	0.158
5	Kapton	1	0.004	0.158
6	FM36	5	0.004	0.158
7	Al 2024	10	4.80	189.0
8	FM36	5	0.004	0.158
9	Al honeycomb 1000	1	0.0516	2.032
10	FM36	5	0.004	0.158
11	Al 2024	10	4.80	189.0
Front surface absorption		0.72	—	—
Front surface emission		0.86	—	—
Back surface emission		0.89	—	—

Table 6 Net heat flow calculation: solar panels

Front	Back	Difference
	Radiation, W/m^2	
460	435	25
	Temperature, °C	
38.4	31.5	6.9

The net conductivity is 62.91 W/(m²K): it is the inverse of the sum of the inverse conductivities of materials 2–11, that is, omitting the coverglass, which is transparent and transmits by radiation, not conduction. Layer 9, the aluminum honeycomb, accounts for most of the result. Assuming the Stefan–Boltzmann law for both front and back surfaces and one-dimensional heat flow through the thin panels, we derive the results shown in Table 6 for a solar flux of 1367 W/m² at 1 AU from the sun, assuming 90 W/m² converted to electricity.

The heat flow from the solar panels increases the total solar force by about 1.8%. Finally, we added a small correction for the W sensor, based on a special personal computer program written for the purpose. Represented as a Fourier series in the elevation angle, we call the result T30, for the X and Z components of solar force, as follows (units are 1×10^{-5} N):

$$
\begin{aligned}
X &= -11.0 \sin B - 0.2 \sin 3B + 0.2 \sin 5B \\
Z &= -11.3 \cos B + 0.1 \cos 3B + 0.2 \cos 5B
\end{aligned}
\tag{2}
$$

Conclusions

Many users will be able to use the T30 formulas given in Eqs. (2) with no correction. The error is expected to be about 2–3% of the total force, and the corresponding error in orbit prediction will be about 3 or 4 m after 12 h, almost entirely in-track, giving about 0.5-m error in pseudorange. For precise geophysical work, this is not acceptable, and we recommend that the leading constants (in sin B and cos B) be adjusted using tracking data, with a time constant of several weeks. The other constants probably should remain fixed, because the a priori shape of the function should be fairly exact. Eclipse seasons require special treatment. As with Block II/IIA, so also with IIR, a Y-bias force may be encountered, with magnitude equal to the total solar force multiplied by the sine of the yaw angle, and this must be estimated from tracking data. For the most precise results, satellites should not be used less than 6 months after launch or during eclipse season.

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